NOTE ON PROF. NICHOLSON'S SINGULAR VALUE OF Π.

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If we regard equation (6), p. 150, as simply equivalent to $\infty . 0 = 2 \div \pi$ it presents no difficulty; but if, on the other hand, the symbols $(1-1)^{\frac{1}{2}}$ and $(1-1)^{-\frac{1}{2}}$ be supposed to stand for the limits of $(1-x)^{\frac{1}{2}}$ and $(1-x)^{-\frac{1}{2}}$ when $x = 1$, the result appears paradoxical, since then the product of these quantities and therefore its limit is equal to unity.

The former is in fact correct for although equation (2) is the result of putting $x = 1$ in the expansion of $(1-x)^n$, (3) is not, since the process of transforming the infinite series into an infinite product is applieable only when $x = 1$. Thus equation (3) means nothing more than that the infinite product in the second member has zero for its limit; in like manner equation (4) means only that the infinite product in its second member has no limit.

Moreover, the product obtained by taking an infinite number of factors from each series may have any value we choose, for this value is a function of the ratio of the infinite numbers of the factors taken from the two series. If p factors be taken from (4) and q factors from (3) the value of the product, when p and q are both infinite but $p \div q = a$, is

$$\frac{a^n \sin n\pi}{n\pi};$$

putting $n = \frac{1}{2}$ and assuming $a = 1$, as implied in the manner of writing Wallis' Theorem, the result becomes $2 \div \pi$.

Note on Experimental Confirmation of Theoretical Deduction, by the Editor.—If a plane surface is ruled with parallel and equidistant lines and a slender rod, the length of which equals the perpendicular distance between two consecutive lines, is thrown at hazard upon the plane, the probability that it will fall across a line is $2 \div \pi$. (See *Mathematical Monthly*, Vol. II, p. 236.)

If we denote this probability by P, we shall have

$$P = \frac{2}{\pi} = \frac{2}{3.14159} = .6366.$$

Hence a rod thrown at hazard upon the plane 10,000 times should fall across a line 6366 times.

At the recent Montreal meeting of the American Association for the Advancement of Science, Prof. Mendenhall exhibited before Section A. of the Association the result of 30,000 experiments which he had performed by